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Numerical simulation of water wave run-up and impact on structures

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Abstract. This study examines the capability of a depth-averaged two-dimensional numerical model for nonlinear shallow water equations. A finite volume Godunov-type numerical model is developed based on a well-balanced formulation of shallow water equations and the HLLC Riemann solver. The comparisons of numerical results with existing theoretical and experimental ones indicate good agreements on free surface elevation, velocity, and pressure. The model is applied to the evaluation of run-up of bores and resulting hydrodynamic impact on coastal structure behind a seawall. It is shown that the numerical model could reproduce the variation of impact forces with different layouts of seawall and structures.

Keywords: Shallow water equations, hydrodynamic impact, Godunov-type model

1. Introduction

In the past, the characteristics of tsunamis have been widely studied by various numerical models for the nonlinear shallow water equations. In the simulations of tsunami behaviors, however, several problems still remain on the modelling of inundation areas, where the propagating wave front accompanies the moving wet-dry interfaces and discontinuous hydraulic jumps, and multi-reflection exist around coastal structures and buildings. Numerical instabilities are often induced by the complicated flow fields. In addition, the grid resolution needed to represent small scale structures on land is usually substantially high compared with that for open seas. In order to improve the stability and accuracy of numerical models on such areas, the Godunov-type scheme is well-suited. In this study, accordingly, a finite volume Godunov-type numerical model is developed for a well-balanced form of shallow water equations. The model is applied to the analysis of the run-up of bores and resulting hydrodynamic impact on coastal structure behind a seawall. The capability of the numerical model is discussed through comparison with existing analytical and experimental results. Some parameter studies are also conducted.

2. Governing equation and numerical scheme

Governing equation

The two-dimensional non-linear shallow water equations are used as the governing equation

$$\frac{\partial \mathbf{q}}{\partial t}+\frac{\partial \mathbf{f}}{\partial x}+\frac{\partial \mathbf{g}}{\partial y}=\mathbf{s} \tag{1}$$

where t = time, x and y = Cartesian coordinates, $\mathbf{q}$ = vector containing the flow variables, $\mathbf{f}$ and $\mathbf{g}$ = flux vectors in the x and y-direction, and $\mathbf{s}$ = vector of source terms given by

$$\mathbf{q}=\begin{bmatrix}\eta & q_x & q_y\end{bmatrix}^{\mathrm{T}} \tag{2}$$

$$\mathbf{f}=\begin{bmatrix}q_x & uq_x+\frac{1}{2}g(\eta^2-2\eta z_b) & uq_y\end{bmatrix}^{\mathrm{T}} \tag{3}$$

$$\mathbf{g}=\begin{bmatrix}q_y & vq_x & vq_y+\frac{1}{2}g(\eta^2-2\eta z_b)\end{bmatrix}^{\mathrm{T}} \tag{4}$$

$$\mathbf{s}=\begin{bmatrix}0 & -\frac{\tau_{bx}}{\rho}-g\eta\frac{\partial z_b}{\partial x} & -\frac{\tau_{by}}{\rho}-g\eta\frac{\partial z_b}{\partial y}\end{bmatrix}^{\mathrm{T}} \tag{5}$$

Adopting the water level as a flow variable, the formulation has well-balanced properties[1].

Finite Volume Godunov-Type Scheme

The numerical scheme is based on a finite volume Godunov-type scheme. A second-order Runge-Kutta method is used to perform time integration. In the predictor step, the intermediate flow variables are computed by

$$\mathbf{q}_{i,j}^{*}=\mathbf{q}_{i,j}^{n}+\Delta t\mathbf{K}_{i,j}(\mathbf{q}^{n}) \tag{6}$$

$$\mathbf{K}_{i,j}=-\frac{\mathbf{f}_{i+1/2,j}-\mathbf{f}_{i-1/2,j}}{\Delta x}-\frac{\mathbf{g}_{i,j+1/2}-\mathbf{g}_{i,j-1/2}}{\Delta y}+\mathbf{s}_{i,j} \tag{7}$$

where i and j = cell indices, Δt = time step, Δx and Δy = cell size in the x and y-directions. In the corrector step, the flow variables are updated by

$$\mathbf{q}_{i,j}^{n+1}=\mathbf{q}_{i,j}^{n}+\frac{1}{2}\Delta t\left[\mathbf{K}_{i,j}(\mathbf{q}^{n})+\mathbf{K}_{i,j}(\mathbf{q}^{*})\right] \tag{8}$$

The numerical fluxes are evaluated from an approximate Riemann solver. First, the Riemann states of flow variables at both sides of the cell interfaces are reconstructed from the cell-centered information by applying the minmod slope limiter. The numerical fluxes are then evaluated by the HLLC (Harten-Lax-van Leer-Contact wave) Riemann solver [2].

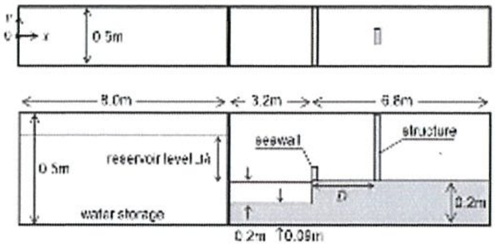

Fig.1 Set up of Arimitu (2012)'s experiment

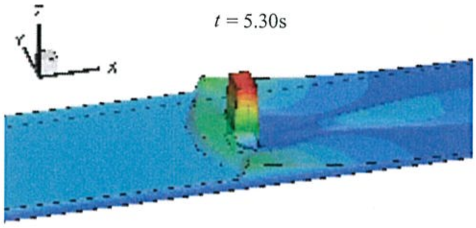

Fig.2 Snapshot of free surface elevation

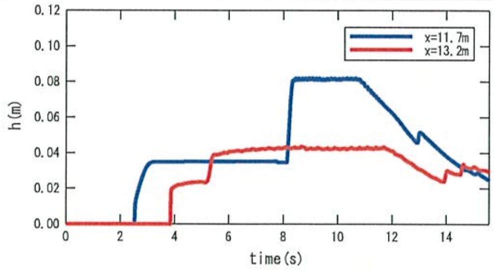

Fig.3 Time history of surface elevation (D=1m)

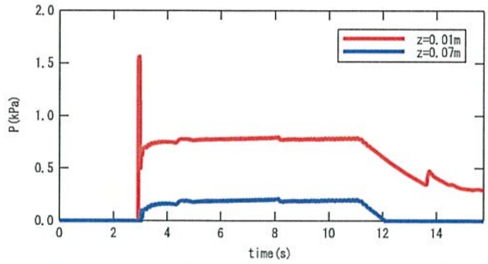

Fig.4 Time History of pressure (D=1m)

3. Numerical Results

First, the numerical model has been tested against two flow problems for a flat bed: the dam-break on wet bottom and on dry bottom. The former includes shock formation, and the latter concerns the wave propagation on dry bed. For both cases, the computed surface elevation and flow velocity accurately reproduced the corresponding analytical solutions [3]. The model is then verified with the analytical solution for long-wave resonance in a circular parabolic basin [4]. Good agreement are obtained, and the model capabilities for computing wave run-up and backwash on sloping bathymetry are verified.

The model is then applied to the problem of tsunami run-up over a seawall, and the numerical results are compared with the hydraulic experiment by Arimitu et al. [5]. The experimental layout is presented in Fig.1. A bore is generated at $t = 0$ by the dam-break technique. The numerical conditions are set as follows: reservoir level $\Delta h = 0.15$ m; seawall height $Hw = 0.02$ m; structure type = rectangular cylinder (0.07×0.1×0.4 m); distance from seawall $D = 0.5$, 1.0, 2.0, 3.0, 4.0 and 5.0 m; Maninng friction coefficient $n = 0.014$ $\mathrm{m}^{-1/3}$. An Example of the computed water surface elevation right after the dam-break wave front hitting the structure is illustrated in Fig. 2. Corresponding time histories of free surface elevation on the centerline (y=0 m) in front of (x=11.7 m) and behind (x=13.2 m) the structure are presented in Fig.3. The pressure on the centerline of the front face of structure is estimated by the following[5] :

$$P(z,t) = \rho g\{h_f - z\} + \rho u_f(t)^2 \tag{9}$$

where P = pressure, ρ = water density, g = gravitational acceleration, h_f= inundation height, z = the location of action, u_f= velocity in x-direction in front of the structure. The time history of pressure acting on two gages at z = 0.01 and 0.07 m are presented in Fig.4.

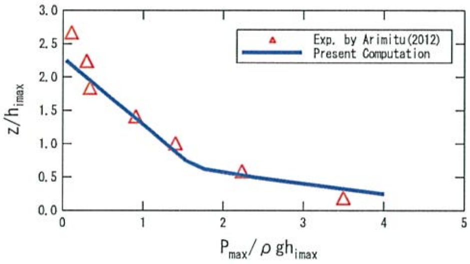

Fig.5 Comparison of maximum pressure between computation and experiment (D=1m)

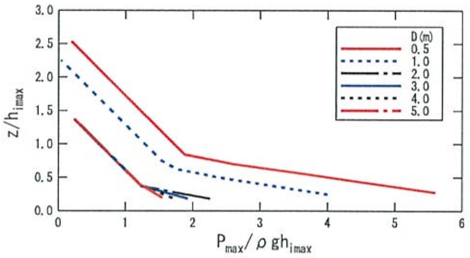

Fig.6 Comparison of maximum pressure for different layouts

The high impact pressure is induced by the collision of supercritical flow close to the bottom ($z = 0.01$ m), while such an instantaneous increase is not recognized at $z = 0.07$ m. Comparison of maximum pressure between experiment and computation is presented in Fig.5. The computational results successfully reproduced the experimental results. The variation of maximum pressure among various layouts of structures is presented in Fig.6. The distance from the seawall is shown to have a strong influence of the maximum pressure.

4. Summary remarks

This study examines the capability of a depth-averaged two-dimensional numerical model for nonlinear shallow water equations. A finite volume Godunov-type numerical model is developed based on a well-balanced formulation of shallow water equations and the HLLC Riemann solver. The model is applied to the evaluation of run-up characteristics of bores and resulting hydrodynamic impact on coastal structure behind a seawall. The comparison with existing experiment show good agreement between numerical and experimental results. The numerical results also indicate the capability of the model to reproduce the variation of flow characteristics with different layouts of seawall and structures.

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