The behavior analysis of Spatial Singular Mode Angle due to addition of noise to the data in an actual bridge experiment

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Abstract. IoT progresses rapidly with the digitization of the world. In civil engineering, it is necessary to digitize the physical space by sensing. Complete IoT using a lot of sensors can realize cost-saving inspection and monitoring for important infrastructure such as bridges.

In the transitional period of IoT, installation of sensors on bridges is cost-labor. On the other hand, there is "On-going Monitoring" that uses a sensor installed on a going vehicle. Spatial Singular Mode Angle (SSMA) shows the possibility of detecting bridge damage as screening index, however, the effect of noise generated in measurement on an actual bridge has not been evaluated enough in previous study. Noise for SSMA can be defined as signal noise or measurement error such as GPS error. Since SSMA can be assumed as a mechanical index based on signal analysis technology, it is an effective for evaluating the features of latent space and inference results of AI.

This study carried out the experiment on four bridges (PC/RC has three and steel has one). The addition of noise is reproduced by addition of random noise or smoothing. The behavior of SSMA in data feature changes are evaluated by these comparative verifications.

Keywords: On-going Monitoring, Vehicle-Bridge Interaction, Spatial Singular Mode Angle, Integration of data-driven and physics-based methods.

1 Introduction

Digital twin on bridge construct the Cyber-Physical System (CPS) by a lot of sensors, for example, accelerometer or UAV's camera. The future inspection will be carried out by the data integration. However, the installation of sensor on the bridge directly is hard, and the cost for detection of the slight influence from structural changes becomes often high now. For instance, change of natural frequency is depended to rigidity of bridge, thus the detection of small damage need extreme accurate and robust sensors if before the rigidity change becomes larger. A lot of sensors is necessary for modal analysis, and the robustness to numerical integration is required for calculation of accurate deflection. They causes the growth of total cost for bridge maintenance, and it can be assumed that the feasibility will become lower. Additionally, it is consider that the power supply and communication system are necessary for realization of long term bridge monitoring. Ones on vehicle doesn't become expensive, and not measure accurately. However, the spec of sensor on vehicle and the index based on the measure data should be validate considering this problem. Yan et al proposed the method to estimate the bridge natural frequency from vehicle vibration with solved the Vehicle-Bridge Interaction (VBI), however, their study didn't consider the road profile [1]. Nagayama et al estimated the natural frequency of bridge with considered the road profile [2]. The bridge natural frequency can't be estimated easily because the effect of damage is small and they is disturbed by measurement noise, thus the cost of sensor becomes high to capture them. On the other hands, the method using mode shape for damage detection is proposed [3-5]. Spatial Singular Mode Angle (SSMA), which is one of the damage index for bridge damage based on estimated mode shape, use the Singular Value Decomposition (SVD), hence the mode is expected to be robust for time space. Continuous SSMA is proposed to detect the structural change [6]. This index is calculated from continuous vehicle vibration data, with shift of a fixed calculation length. However, the amplitude of bridge response is smaller than the vehicle one, the estimated mode shape is often affected from quantization bit rate of analog-digital converter (ADC). Previous study evaluated the effect to frequency or SSMA from the difference in bitrate of ADC [7]. The vehicle put two sensor systems which has 17 or 23 bit resolution is used for measurement experiment on three actual bridge, and the frequency analysis and calculation SSMA based on the data measured by both of sensor systems. The Use case in a practice, an electrical or mechanical noise can be considered. SVD is robust to time domain noise because of their calculation structure, however, the effect for SSMA by their noise is uncertain. This study smooths or adds a noise to the data on 17 bit ADC, and their behavior of SSMA is validated.

2 Vehicle-Bridge Interaction Theorem

The mathematical theorem of SSMA is described. Calculation needs vibration and position at the front, rear axles of vehicle. The vibration should be obtained from mass points under the spring. The assumed vehicle (Half Car) model is shown in Figure 1. This model has a rigid body as sprung-mass system, of which mass is m_s, and of which inertia moment is I_s. The point G indicates the centre of gravity, and the distances from the point G to the front and the rear axles are L_1 and L_2, respectively. In this figure, it is noted that L_1 and L_2described as if as equal, however they are ordinary different because the engine often put near front wheel. The subscript i (= 1, 2) represents the front and rear axles. $z_{si}(t)$ and $z_{ui}(t)$ are the vertical displacements of the sprung-mass and the unsprung-mass. $u_i(t)$ is the forced displacement input under the i-th axle. k_{si} and c_{si} are the spring stiffness and the damping of the spring-mass at the i-th axle. m_{ui}, k_{ui}

and c_{ui} are the mass, spring stiffness and damping of the unsprung-mass at the i-th axles, respectively. The equation of motion of the vehicle can be described by the following.

$$\mathbf{M}_{\mathrm{V}}\ddot{\boldsymbol{z}}(t)+\mathbf{C}_{\mathrm{V}}\dot{\boldsymbol{z}}(t)+\mathbf{K}_{\mathrm{V}}\boldsymbol{z}(t)=\mathbf{C}_{\mathrm{P}}\dot{\boldsymbol{u}}(t)+\mathbf{K}_{\mathrm{P}}\boldsymbol{u}(t) \tag{1}$$

respectively. ($\dot{}$) and ($\ddot{}$) denote the first-order and second-order time differentiation.

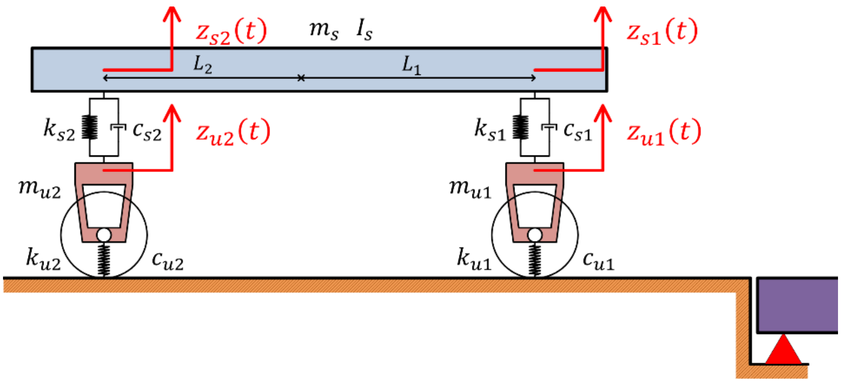


Figure 1. Vehicle (Half Car) Model.

Since the number of sensors are same with that of estimated mode shapes, when we set a sensor on each axle, only the first and second modes can be obtained. When we use only lower mode shapes, their variation can be explained only from two factors: the measurement environment and the structural change. The latter is, in other word, a damage. On the other hand, when we use more sensors, the main factor of variation becomes the ill condition problem, which means that the results depend only on noise, not on the status of the structure.

On the other hand, the bridge displacement at position x and time t can be decomposed as follows:

$$y(x,t)=\sum_{k}\phi_k(x)q_k(t) \tag{2}$$

$\phi_k(x)$ is the k-th order mode shape, and $q_k(t)$ is the k-th order basis coordinates. Substituting each axle position $x_i(t)$ into Equation 2, the bridge displacement just under the i-th axle is shown below:

$$y_i(t)=\sum_{k}\phi_k\big(x_i(t)\big)q_k(t) \tag{3}$$

Assuming that the road roughness at the position of x is $R(x)$, the input component of the i-th axle of the vehicle at the time of t is shown below:

$$r_i(t) = R\big(x_i(t)\big) \tag{4}$$

Then, the forced displacement inputs can be described by

$$\boldsymbol{u}(t) = \boldsymbol{y}(t) + \boldsymbol{r}(t) \tag{5}$$

Equation 5 can be rewritten in

$$\boldsymbol{u}(t) = \boldsymbol{\Phi}(t)\boldsymbol{q}(t) + \boldsymbol{r}(t) \tag{6}$$

Next, $\phi_k\big(x_i(t)\big)$ can be discretized by interpolation as below:

$$\phi_k(x) = \sum_{\boldsymbol{j=1}}^{\boldsymbol{n}} a_{jk} N_j(x) \tag{7}$$

When the base function $N_j(x)$ is the Lagrangian function, the coefficient a_{kj} indicates the amplitude of k-th order mode shape at the discretized position x_j. Figure 2 shows the concept of this interpolation.

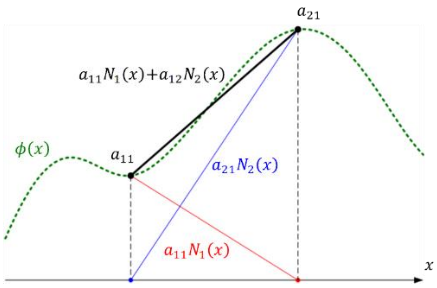


Figure 2. Concept of interpolation.

By using matrix expression, Equation 7 becomes

$$\boldsymbol{\Phi}(t) = \mathbf{N}(t)\mathbf{A} \tag{8}$$

where the (k, j) component of the matrix $\mathbf{A}$ is a_{kj}. Assuming that the unsprung-mass parameters of the front and rear axles are same, which means that $k_{u1}/m_{u1} = k_{u2}/m_{u2} = k_u/m_u$ and $c_{u1}/m_{u1} = c_{u2}/m_{u2} = c_u/m_u$, the vertical acceleration vibrations of the unsprung-mass can be described by

$$\ddot{\boldsymbol{z}}_u(t) = \begin{Bmatrix} \ddot{z}_{u1}(t) \\ \ddot{z}_{u2}(t) \end{Bmatrix} = \mathbf{N}(t)\mathbf{A}\boldsymbol{\sigma}(t) + \bar{\boldsymbol{\epsilon}}(t) \tag{9}$$

If the position of each axle $x_j(t)$ are available, the interpolation matrix $\mathbf{N}(t)$ can be calculated. Since the unsprung-mass vibrations $\ddot{\boldsymbol{z}}_u(t)$ and the interpolation matrix $\mathbf{N}(t)$ are known, we obtain

$$\mathbf{N}^{-1}(t)\ddot{\boldsymbol{z}}_u(t) = \mathbf{A}\boldsymbol{\sigma}(t) + \boldsymbol{\epsilon}(t) \tag{10}$$

$$\boldsymbol{\epsilon}(t) = \mathbf{N}^{-1}(t)\bar{\boldsymbol{\epsilon}}(t) \tag{11}$$

The left side of Equation 10 is the spatial correction of vehicle vibrations. Based on Equation 10, the mode shape $\mathbf{A}$ can be estimated by SVD of $\mathbf{N}^{-1}(t)\ddot{\boldsymbol{z}}_u(t)$. By SVD, the mode shape $\mathbf{A}$ and the bridge vibration component $\boldsymbol{\sigma}(t)$ are calculated at the same time. The bridge components includes only information about the bridge vibration and unsprung-mass characteristics of the vehicle. Others are included in the error term $\boldsymbol{\epsilon}(t)$: the vehicle responses: $\boldsymbol{z}(t)$, $\dot{\boldsymbol{z}}(t)$ and the road roughness: $\boldsymbol{r}(t)$ and $\dot{\boldsymbol{r}}(t)$. Since $\mathbf{N}^{-1}(t)\ddot{\boldsymbol{z}}_u(t)$ is time function, it can be described as data matrix $\mathbf{D} \in R^{2\times T}$. T means the number of the measured data. The SVD of $\mathbf{D}$ is described by the product of an orthogonal matrix $\mathbf{U} \in R^{2\times 2}$, a diagonal matrix $\boldsymbol{\Sigma} \in R^{2\times 2}$ and an orthogonal matrix $\mathbf{V} \in R^{T\times 2}$ ($\mathbf{V}^{\mathrm{T}}\mathbf{V} = \mathbf{I}$: the unit matrix) as below:

$$\mathbf{D} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathrm{T}} \tag{12}$$

where $\mathbf{U}$ is the estimation of $\mathbf{A}$, and $\boldsymbol{\Sigma}\mathbf{V}^{\mathrm{T}}$is the estimation of $\boldsymbol{\sigma}(t)$ in the form of data matrix. In order for SVD of $\mathbf{D}$ to accurately estimate the bridge mode shape $\mathbf{A}$, the following conditions need to be satisfied: $\boldsymbol{\sigma}(t)$ is uncorrelated and the error term $\boldsymbol{\epsilon}(t)$ is white noise. The bridge vibration components $\boldsymbol{q}(t)$ and $\dot{\boldsymbol{q}}(t)$ are transient responses induced by the traffic loads, in this case. Thus, it is considered that the real values of $\boldsymbol{\sigma}(t)$ does not satisfy the condition of a). While the SVD process gives the estimated bridge vibration components $\boldsymbol{\Sigma}\mathbf{V}^{\mathrm{T}}$, they are just uncorrelated signals near $\boldsymbol{\sigma}(t)$. The error due to this affects on the estimated mode shape $\mathbf{U}$. This means that the estimation mode shape $\mathbf{U}$ and the succeeding index SSMA deviate slightly from the correct mode shape. This effect on SSMA, however, can be expected to be unchanging under the same measurement condition. Generally, a local damage on a bridge never influence the dynamic indices of the global system of the structure. Thus, it is expected that $\mathbf{A}$ remains unchanged even after the damage. However, because the local bridge responses are easily affected by the damage, the component $\boldsymbol{\sigma}(t)$ changes. The estimation for it is $\boldsymbol{\Sigma}\mathbf{V}^{\mathrm{T}}$ and it cannot trace the transition. This error is included in the error of $\mathbf{U}$. This is the mechanism of SSMA to react a bridge's local damage.

3 Experiment On Actual Bridge

3.1 The Verification Of Acceleration And The Frequency

Experiment is carried out on PC and Steel bridges. They are three PC and one steel bridge and called as PC1-3 and S1. Acceleration and their power spectrum (PS) on 17 bit rate ADC are shown in Figure. 3. The vehicle vibration over bridge is identified from GPS position of sensor on vehicle and bridges entrance and exit. Blue shows the front un-sprung z axis vibration, and orange shows the rear one. Gravity direction is negative. SSMA uses the unsprung vibration, thus this study focuses them. The bridges and vehicle parameter are shown in Table 1. The velocity of vehicle is decided by actual traffic speed. Other vehicles which go through the same bridge then is ignored be-cause the experiment vehicle weight is very heavy (13.8t) and it is confirmed that the similar weight vehicle didn’t go with the experiment vehicle when the data in this study are measured. All bridges are evaluated as level I by Japanese bridge inspection expert. The sensor position on vehicle is shown in Figure. 3. All PS have highest peak on under 5Hz. Since the peaks is independent for bridge length, the result can be caused by vehicle such as engine. The shorter bridge often has around 10~30 Hz as natural frequency, thus the result is proper about PC1-2 in Figure .4(a)-(d). Notice the short PC bridge are ordinary more rigid than similar steel ones. PC3 and S1 have also a peak around 10 Hz, and their trend are different, respectively in Figure 4(e)-(h). Since this study doesn’t focus on the change of frequency and their value, this verification only confirms the sensor behavior from these result.

Table 1. The parameter in Experiment

	PC1	PC2	PC3	S1
Span [m]	12.6	14	30.88	30
Girder Type, Number	I	T, 4	T, 4	Steel, I, 4
Vehicle Weight [t]		13.8		
Vehicle Velocity [km/h]	17.3	43.6	27.1	30.0

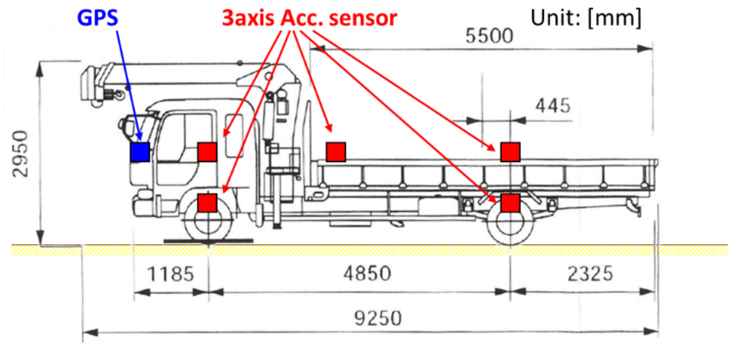
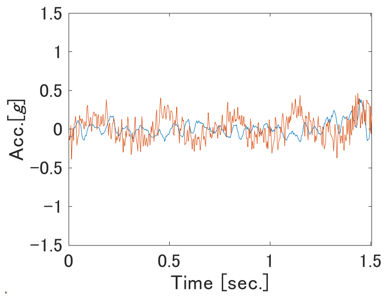
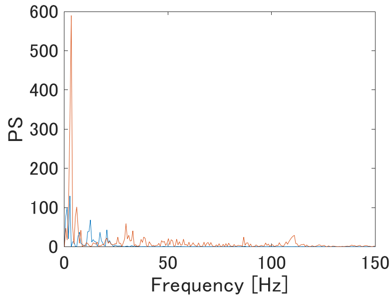


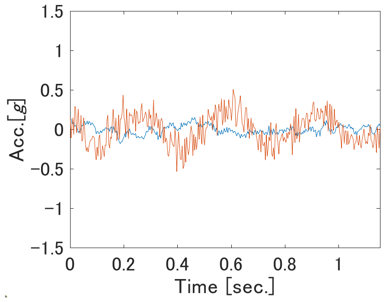
Figure 3. The position of sensor on vehicle



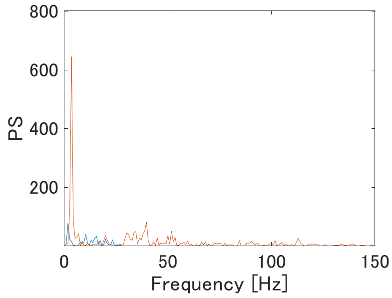
(a) PC1 Acceleration



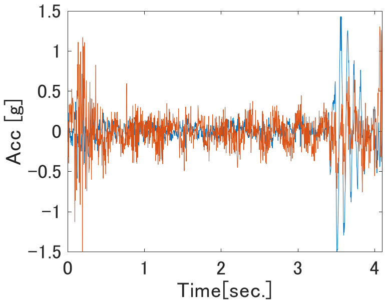
(b) PC1: PS



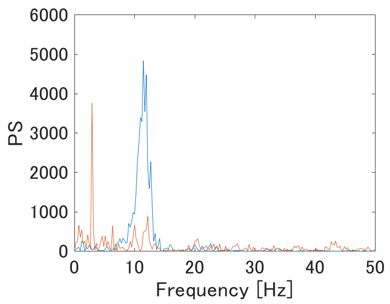
(c) PC2 Acceleration



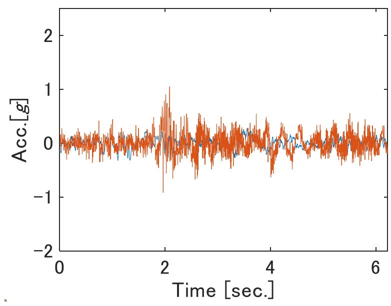
(d) PC2: PS



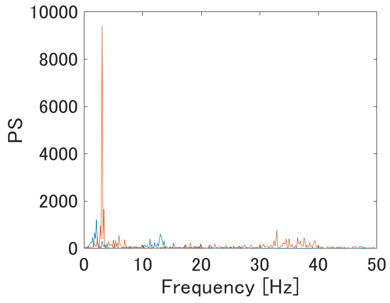
(e) PC3 Acceleration



(f) PC3: PS



(g) S1 Acceleration

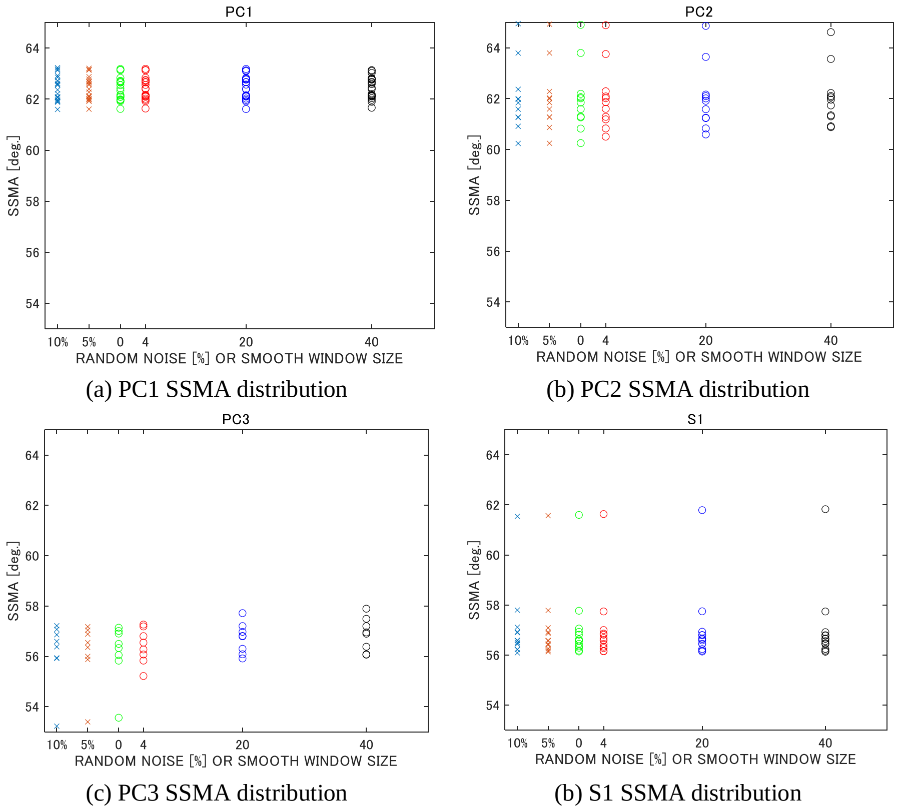


(h) S1: PS

Figure 4. Acceleration and Power Spectrum of Un-sprung Vehicle Vibration.

4 SSMA Calculation From Original Or Smoothed Data

SSMA is calculated from acceleration with adding random noise or smooth. The number of measurement are over 10 times for each bridge. Random noise is added +-5% of measured acceleration. Smoothing is carried out by Gaussian filter (GF). GF is assumed as low-pass filter. The window size is changed to 4, 20 and 40. The result is shown in Figure. 5. Green circle is SSMA from the original data. Crosses show the result of noised ones. A red, blue and black circle shows the result of smoothed ones. The distribution of SSMA is different PC1,2 with PC3,S1. PC1,2 has 12.6[m] or 14 [m] length, and PC3,S1 has 30[m] or 30.88[m] length. It can be assumed that large noise rate causes the variance is larger, and large window size causes the variance is smaller. This assumption appears in Table 2 except with S1. S1 is located where the vehicle is likely to brake, thus the change of vibration data can affect the result. In addition, the difference of structure (i.e. PC and Steel) should be considered. Next, the skewness is validated in Table 3. When skewness is 0, the distribution is closed to normal and the result is validated based on normal distribution assumption.



(a) PC1 SSMA distribution

(b) PC2 SSMA distribution

(c) PC3 SSMA distribution

(b) S1 SSMA distribution

Figure 5. SSMA with adding a random noise or smoothing for each bridge.

Table 2. Variance change of SSMA with GF smoothing

	Random Noise		Window Size			
Bridge	10%	5%	0	4	20	40
PC1	0.231	0.214	0.200	0.194	0.197	0.180
PC2	1.764	1.752	1.739	1.648	1.562	1.273
PC3	1.627	1.467	1.321	0.485	0.362	0.437
S1	2.043	2.058	2.076	2.100	2.217	2.249

Table 3. Skewness change of SSMA with GF smoothing

	Random Noise		Window Size			
Bridge	10%	5%	0	4	20	40
PC1	0.223	0.188	0.147	0.097	0.047	-0.011
PC2	1.030	1.027	1.023	1.102	1.172	1.169
PC3	-1.663	-1.627	-1.582	-0.300	0.164	0.113
S1	2.665	2.690	2.710	2.736	2.778	2.789

Table 4. Kurtosis change of SSMA with GF smoothing

	Random Noise		Window Size			
Bridge	10%	5%	0	4	20	40
PC1	1.883	1.952	2.027	2.034	1.963	1.894
PC2	3.411	3.404	3.392	3.413	3.591	3.524
PC3	4.657	4.572	4.461	2.106	2.064	1.813
S1	9.017	9.113	9.193	9.301	9.462	9.506

The skewness for PC1,2 is closed to 0 with reduction of noise and smoothing carried out, and ones for PC3,S1 is not. Therefore, the change of SSMA distribution with noise or smoothing corresponds to the bridge feature respectively, and it is expected that bridge damage can be detected sensitively by validation of their value. In Table 4, the kurtosis is validated. The kurtosis of normal distribution is 0 in this study. It is also expected that the bridge damage can be detected sensitively with higher kurtosis. The kurtosis for PC1,2 is changed hardly. Ones for PC3 is decreased with smoothing, and ones for S1 is tend to be increased. It is suggested that the sensitivity became high for steel bridge because the steel ones can vibrate easily, however, the effect of vehicle braking should be also considered. Thus, the analysis for each signal feature value or other bridge data should be necessary. This result can be affected from the data length because data for longer bridge on same vehicle speed became long and the effect for unit data by addition of noise or smoothing become small. Therefore, the analysis for signal feature value of the data should be carried out where the environment of location, the type of bridge and data length are considered.

5 Conclusion

This study reveals the mechanism and the robustness of SSMA, which is an index of bridge health, by verification of the behavior under the addition of a noise or smoothing. The findings are shown as below:

1. On PC3, it is appeared that the distribution converges to normal distribution along with making strong the smoothing. The result of S1 may have been affected by the environment of location, however, the result is not similar to PC3. This result suggests the effect by the type of bridge, and the interaction to SSMA should be validated by big data analysis.

2. PC1,2 which has a short span, has a little change with the addition of noise or smoothing, and PC3 has a large effect from them. It is suggested that the result depends on the data length.

As future works, they are considered that the verification by the signal processing including the numerical simulation, and the statistical analysis for the big data.

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